Vibrational Superposition States Without Rotating Wave Approximation

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Abstract

We propose a scheme to generate superpositions of coherent states for the vibrational motion of a laser cooled trapped-ion. It is based on the interaction with a standing wave making use of the counter-rotating terms, i.e. not applying the rotating wave approximation. We also show that the same scheme can be exploited for quantum state measurement, i.e. with the same scheme non-classical states may be reconstructed.

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1 Introduction

In recent years there has been great interest in quantum state preparation and measurement [1]. In particular the generation of non-classical states was proposed for light field [2] and for vibrational motion of a trapped ion [3]. Consequently, several schemes to reconstruct such states have been developed [4, 5, 6]. In both cases (field and ion) key features are shared for their generation and reconstruction, e.g. the relation of the atomic inversion with quasiprobability distributions as they both can be expressed as a sum of diagonal density matrix elements (see for instance [7, 8]). On the other hand, both systems, atom-field and ion-laser interactions can be described by the same kind of Hamiltonians under certain conditions (Lamb- Dicke or intensity approximations [3]). Here we will concentrate on the vibrational motion of an ion, bearing in mind that, because of the similarity of such a system with the atom-field interaction, the scheme could be extended to the later case. It was recently shown by Wilkens and Meystre [9] that the use of the Rotating Wave Approximation (RWA) (in the atom-field interaction case) does not permit the possibility of many absorption and/or emission processes, and therefore it is needed a second mode of the electrmagnetic field to change the situation: in such a case, although an absorption process is followed by an emission process, there exist situations in which absorption from the first mode may be followed by emission into the second mode, then again by absorption from the first mode, etc. This allows coherences between distant Fock states to take place. Here we show that by not making use of the RWA, the ion effectively becomes sensitive to coherences between such distant Fock states giving rise to Schrödinger cat-like states [10]. Moreover, we show that the same mechanism allows the reconstruction of vibrational state.

2 The model

Let us consider a trapped ion laser cooled into the Lamb-Dicke limit using a strong transition and located at the node of an optical standing wave (SW). Such system can be

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described by the Hamiltonian [5] (in a frame rotating at the SW frequency $\omega_L/2\pi$)

$$\hat{H} = \hbar\nu\hat{n} + \hbar\Delta\hat{\sigma}_z + \hbar\eta\Omega\hat{X}\hat{\sigma}_x, \tag{1}$$

where ν is the oscillation angular frequency of the ion in the trap, $\Delta = \omega_0 - \omega_L$, is the detuning between the atomic (ω_0) and the SW angular frequencies, Ω is the Rabi frequency for the two-level transition, η is the Lamb- Dicke parameter ($\eta << 1$). $\hat{n} = \hat{a}^\dagger\hat{a}$ is the number operator and $\hat{X} = \hat{a}^\dagger + \hat{a}$ is the generalized position operator, with $\hat{a}^\dagger$ and $\hat{a}$ the creation and annihilation operators respectively for the vibrational mode. $\hat{\sigma}_z$ is the inversion operator and $\hat{\sigma}_x = \hat{\sigma}_+ + \hat{\sigma}_-$ is the polarization operator, where $\hat{\sigma}_+$ and $\hat{\sigma}_-$ are the atom raising and lowering operators, respectively. By considering the resonant case, i.e. $\Delta = 0$, from the Hamiltonian (1) we can obtain the evolution operator in the following form (see for instance [11])

$$\hat{U}(\tau) = e^{i\phi(\tau)}e^{-i\hat{n}\tau}\hat{D}[\alpha(\tau)\hat{\sigma}_x], \quad \tau = \nu t, \tag{2}$$

where we have defined

$$\hat{D}[\alpha(\tau)\hat{\sigma}_x] = \exp\left\{\hat{\sigma}_x[\alpha(\tau)\hat{a}^\dagger - \alpha^*(\tau)\hat{a}]\right\}, \tag{3}$$

and

$$\phi(\tau) = \eta^2\frac{\Omega^2}{\nu^2}(\tau - \sin\tau), \quad \alpha(\tau) = \eta\frac{\Omega}{\nu}(1 - e^{i\tau}). \tag{4}$$

Let us now consider the generic initial state

$$|\Psi(0)\rangle = |\psi_e\rangle|\psi_v\rangle, \tag{5}$$

where the atomic initial state is

$$|\psi_e\rangle = \mathcal{A}|g\rangle + \mathcal{B}e^{i\varphi}|e\rangle, \tag{6}$$

with φ, $\mathcal{A}$, and $\mathcal{B}$ real numbers such that $\mathcal{A}^2 + \mathcal{B}^2 = 1$; $|e\rangle$ and $|g\rangle$ represent two different states of the atom. Then, the state (5) evolves according to

$$|\Psi(\tau)\rangle = e^{i\phi(\tau)}e^{-i\hat{n}\tau}\left\{\mathcal{A}\left[\cosh\left(\alpha(\tau)\hat{a}^\dagger - \alpha^*(\tau)\hat{a}\right)|g\rangle + \sinh\left(\alpha(\tau)\hat{a}^\dagger - \alpha^*(\tau)\hat{a}\right)|e\rangle\right]\right.$$

$$\left. + e^{i\varphi}\mathcal{B}\left[\sinh\left(\alpha(\tau)\hat{a}^\dagger - \alpha^*(\tau)\hat{a}\right)|g\rangle + \cosh\left(\alpha(\tau)\hat{a}^\dagger - \alpha^*(\tau)\hat{a}\right)|e\rangle\right]\right\}|\psi_v\rangle. \tag{7}$$

3   Generation of superpositions of coherent states

To this end, we consider the ion initially to be not excited, $|\psi_e\rangle = |g\rangle$, and the vibrational motional state to be a coherent state $|\psi_v\rangle = |\alpha_0\rangle$. From Eq.(7), we easily get the evolved state. For particular times, $\tau = (2q + 1)\pi$ with q integer number, we obtain

$$|\Psi(\tau)\rangle = \frac{1}{2}\left\{\left[|-(\alpha_0 + \alpha)\rangle + |\alpha - \alpha_0\rangle\right]|g\rangle + \left[|-(\alpha_0 + \alpha)\rangle - |\alpha - \alpha_0\rangle\right]|e\rangle\right\}, \quad (8)$$

where $\alpha = -2\eta\Omega/\nu$. We have neglected the overall phase ϕ since it is not relevant. It is now immediate to see that an atomic selective measurement [12] yields a superposition of two distinct coherent states by means of the wave function collapse. In particular, for $\alpha_0 = 0$, it is possible to get the even or odd coherent states [13] depending on the result g or e of the measurement. Furthermore, if we allow for more than one interaction, superpositions of coherent states in a line can be produced, i.e. any non-classical state can be produced [14].

4   Vibrational State Measurement

Let us now consider the reconstruction method within the same scheme. In this case the state $|\psi_v\rangle$ would be unknown. To be more general, instead of a pure state $|\psi_v\rangle$, we are going to consider an initial vibrational state $\hat{\rho}_v$. Under such assumptions, we can calculate the probability of the ion being in the ground state again from Eq. (7), which can be written as

$$P_g = \frac{1}{2} + \frac{\mathcal{A}^2 - \mathcal{B}^2}{2}\mathrm{Tr}_v\left\{\cos\left(k\hat{X}_\theta\right)\hat{\rho}_v\right\} - \mathcal{A}\mathcal{B}\sin\varphi\mathrm{Tr}_v\left\{\sin\left(k\hat{X}_\theta\right)\hat{\rho}_v\right\} \quad (9)$$

where we have introduced the quadrature

$$\hat{X}_\theta = \hat{a}e^{-i\theta} + \hat{a}^\dagger e^{i\theta} \quad (10)$$

and the radial and angular variables

$$k = 2|\alpha(\tau)| = 4\eta\frac{\Omega}{\nu}\sin\left(\frac{\tau}{2}\right), \quad \theta = \arctan\left[\frac{\sin\tau}{1 - \cos\tau}\right] - \frac{\pi}{2} \quad (11)$$

Eq. (9) shows that the probability of the ion being in its ground state is proportional to the characteristic function [15] of the vibrational motion of the ion

$$\chi(k,\theta) = \mathrm{Tr}_v \left\{ e^{ik\hat{X}_\theta} \hat{\rho}_v \right\} \qquad (12)$$

with the property $\chi(k,\theta+\pi) = \chi(-k,\pi)$. This function contains all information about the state [15, 16] which makes Eq. (9) a significant result. It can also be related to the Shirley (or ambiguity) function which is the totally off-diagonal complement of the Wigner function [16]. Hence, by appropriately adjusting φ, $\mathcal{A}$ and $\mathcal{B}$ it is possible to measure both the even and the odd part of the characteristic function.

For what concerns the argument k, it can be varied by changing the Rabi frequency Ω, i.e. by changing the intensity of the standing wave. Instead, the quadrature phase θ, can be varied by means of the interaction time τ; but because the latter affects also k, it is preferable to set the desired quadrature phase θ with a free evolution prior the interaction, keeping τ equal for all sets of measurements. Moreover, there exist standard simple techniques to monitor the probability of the ion being in its ground (excited) state (see [5, 6]). The measurement of χ allows in reality the direct sampling of density matrix elements in some representations since we can write

$$\hat{\rho}_v = \int_{-\infty}^{\infty} \int_0^{\pi} \frac{d\theta}{\pi} \chi(k,\theta) \, K(\hat{X}_\theta), \qquad K(\hat{X}_\theta) = \frac{|k|}{4} \exp\left[-ik\hat{X}_\theta\right], \qquad (13)$$

and the kernel operator K results bounded e.g. in number representation [17]. Instead, direct sampling of the Wigner function is not possible since the corresponding kernel is not bounded. The analogous problem exist in optical homodyne tomography, where the measured quantity is the Fourier transform of the characteristic function [18].

5   conclusions

We have considered a trapped ion interacting with a radiation field, and we have shown that by exploiting counter-rotating terms it is possible to produce and retrieve quantum coherences in the vibrational degree of freedom. The significance of our method relies on

its simplicity, as we do not need to do any further assumptions but the standard one of small Lamb-Dicke parameter.

Acknowledgments

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